Introduction to Statistics — Quiz #2 (60 minutes)

April 7, 2026 (Tuesday)

Section: ________ Cadet Number: ______________ Name: ________________ Score: _____________

- All solutions must include a detailed step-by-step explanation.
- If an answer has more than four decimal places, round to the **fourth decimal place**.
- **Reference table** is provided on the last page of the exam.

1. Suppose that 60% of students at a university hold a driver's license. 10 students are randomly selected, and X denotes the number of students among them who hold a driver's license. Answer the following questions. [25 points]

(1) Find the probability that exactly 8 or exactly 9 of the selected students hold a driver's license.

Solution: Since X is the number of license holders in $n = 10$ trials with success probability $p = 0.6$, we have $X \sim B(10,\, 0.6)$. Therefore,

$$P(X=8)+P(X=9) = \binom{10}{8}(0.6)^8(0.4)^2 + \binom{10}{9}(0.6)^9(0.4)^1 \approx \boxed{0.1612}.$$

(2) Find the expectation and variance of the random variable X.

Solution: For $X \sim B(n, p)$, we have $E(X) = np$ and $\mathrm{Var}(X) = np(1-p)$. Since $X \sim B(10,\, 0.6)$,

$$E(X) = 10 \times 0.6 = \boxed{6}, \qquad \mathrm{Var}(X) = 10 \times 0.6 \times 0.4 = \boxed{2.4}.$$

2. Read each statement below and determine whether it is **True (O)** or **False (X)**, or fill in each blank with an appropriate term chosen from the word box below. [15 points]

Bernoulli trial	Bernoulli distribution	binomial distribution	uniform distribution
population	parameter	law of large number	random sample
statistic	sampling distribution	central limit theorem	distribution of sample mean
normal distribution	standard normal distribution	estimation	distribution of sample variance
degrees of freedom	sample proportion	χ^2-distribution	random variable

(1) Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2. The sample mean $\bar{X}$ follows a binomial distribution with mean μ and variance σ^2/n. (O / X) ________

(2) A collection of n independent random variables drawn from the same distribution is called a (____________________) of size n.

(3) When a random variable X has the following probability density function

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b, \\ 0, & \text{otherwise}, \end{cases}$$

X is said to follow a (____________________) on the interval (a, b).

(4) A function of a random sample that does not involve any unknown parameters is called a statistic, and the probability distribution of a statistic is called a (____________________).

(5) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a population with mean μ and variance $\sigma^2 < \infty$. Then the distribution of $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ can be approximated to the standard normal distribution as n goes to infinity. (O / X) ________

Solution: X / random sample / uniform distribution / sampling distribution / O

3. At a factory, the weight (in grams) of fertilizer produced by Machine A follows a normal distribution with mean $100\,\mathrm{g}$ and standard deviation $3\,\mathrm{g}$, while the weight of fertilizer produced by Machine B follows a normal distribution with mean $95\,\mathrm{g}$ and standard deviation $4\,\mathrm{g}$. One product is randomly selected from each machine. Let X denote the weight (in grams) of fertilizer from Machine A and Y denote the weight (in grams) from Machine B. Find $P(X \geq Y + 17.5)$. [30 points]

Solution: We have $X \sim N(100,\, 3^2)$ and $Y \sim N(95,\, 4^2)$.
We wish to find $P(X \geq Y + 17.5) = P(X - Y \geq 17.5)$. Consider $X - Y$:

$$E(X-Y) = E(X) - E(Y) = 100 - 95 = 5.$$

Since X and Y are independent,

$$\mathrm{Var}(X-Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) = 3^2 + 4^2 = 25 = 5^2.$$

Therefore $X - Y \sim N(5,\, 5^2)$. Standardizing, $Z = \dfrac{(X-Y)-5}{5} \sim N(0,1)$. Thus,

$$P(X - Y \geq 17.5) = P\left(\frac{(X-Y)-5}{5} \geq \frac{17.5-5}{5}\right) = P\left(Z \geq \frac{12.5}{5}\right)$$
$$= P(Z \geq 2.5) = 1 - P(Z < 2.5) = 1 - 0.9938 = \boxed{0.0062}.$$

4. The battery life of portable radios produced by a company follows a normal distribution with mean 45 hours and standard deviation 3 hours. When 16 units are randomly selected, find the probability that the sample mean battery life is in between 44 hours and 47 hours. (Be sure to define your random variable before solving.) [30 points]

Solution: Let X denote the battery life (in hours) of a portable radio produced by the company.
The population follows a normal distribution with mean $\mu = 45$ and standard deviation $\sigma = 3$, so $X \sim N(45,\, 3^2)$.
For a random sample of size $n = 16$,

$$E(\bar{X}) = \mu = 45, \quad \mathrm{Var}(\bar{X}) = \frac{3^2}{16} = \left(\frac{3}{4}\right)^2,$$

so $\bar{X} \sim N\big(45,\, 0.75^2\big)$. Standardizing, $Z = \dfrac{\bar{X} - 45}{0.75} \sim N(0,1)$. Therefore,

$$P(44 < \bar{X} < 47) = P\left(\frac{44-45}{0.75} < \frac{\bar{X}-45}{0.75} < \frac{47-45}{0.75}\right)$$
$$= P(-1.3333 < Z < 2.6667)$$
$$= P(Z < 2.6667) - P(Z < -1.3333)$$
$$= P(Z < 2.6667) - \big(1 - P(Z < 1.3333)\big)$$
$$= 0.9962 - (1 - 0.9088) = \boxed{0.9050}.$$

Reference Table Let $Z \sim N(0,1)$ and $V \sim \chi^2(15)$.

$P(Z<0)=0.5$	$P(Z<0.5)=0.6915$	$P(Z<1)=0.8413$	$P(Z<1.3333)=0.9088$
$P(Z<1.5)=0.9332$	$P(Z<2)=0.9772$	$P(Z<2.5)=0.9938$	$P(Z<2.6667)=0.9962$
$z_{0.0038}=2.6667$	$z_{0.0912}=1.3333$	$z_{0.0062}=2.5$	$z_{0.0228}=2$
$P(V \leq 11.6667)=0.2959$		$P(V \leq 22.6667)=0.9085$	