

# Introduction to Statistics - Quiz #1 (60 minutes)

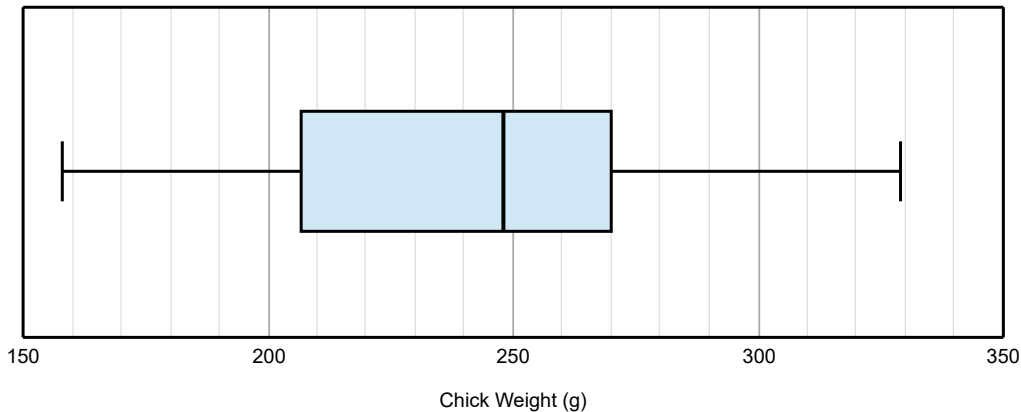
March 19, 2026 (Thursday)

Section(교반): \_\_\_\_\_ Cadet Number(교번): \_\_\_\_\_ Name(성명): \_\_\_\_\_ Score: \_\_\_\_\_

- All solutions must include a detailed step-by-step explanation.
- If an answer has more than four decimal places, round to the **fourth decimal place**.

1. The table below shows descriptive statistics for chick weights (in grams) when soybean meal was used as a feed additive. Based on these values, draw a boxplot for chick weights in the blank graph provided. [30 points]

Minimum	$Q_1$	Median ( $Q_2$ )	$Q_3$	Maximum	Mean
158	206.75	248.0	270.00	329	246.43



2. A company installs security cameras at its building. The probability that a camera's sensor is defective is 0.03. When a camera runs all day, a camera with a defective sensor has a 0.8 probability of losing its picture, while a camera with a normal sensor has only a 0.04 probability of losing its picture. Answer the following questions. [30 points]

(1) Find the probability that a randomly chosen camera loses its picture during operation. (Define all events clearly before solving.)

Solution: Let  $A$  be the event that a camera's sensor is defective, and  $B$  be the event that the camera loses its picture. Then

$$P(A) = 0.03, \quad P(A^c) = 0.97, \quad P(B | A) = 0.8, \quad P(B | A^c) = 0.04.$$

By the law of total probability,

$$\begin{aligned} P(B) &= P(A) P(B | A) + P(A^c) P(B | A^c) \\ &= 0.03 \times 0.8 + 0.97 \times 0.04 = 0.024 + 0.0388 = 0.0628. \end{aligned}$$

(2) Find the conditional probability that a randomly chosen camera has a defective sensor, given that it has lost its picture.

Solution: Since  $A$  and  $A^c$  form a partition of the sample space, by Bayes' theorem,

$$\begin{aligned} P(A | B) &= \frac{P(A) P(B | A)}{P(A) P(B | A) + P(A^c) P(B | A^c)} \\ &= \frac{0.03 \times 0.8}{0.03 \times 0.8 + 0.97 \times 0.04} = \frac{0.024}{0.0628} \approx 0.3822. \end{aligned}$$

3. The joint probability mass function of two random variables  $X$  and  $Y$  and its table are given below. Answer the following questions. [40 points]

$$f(x, y) = \begin{cases} \frac{x+y}{9}, & x = 0, 1, 2, y = 0, 1 \\ 0, & \text{otherwise} \end{cases}$$

$x \backslash y$	0	1
0	0	1/9
1	1/9	2/9
2	2/9	3/9

(1) Find the marginal probability mass function  $f_X(x)$  of  $X$ .

Solution: Using  $f_X(x) = \sum_{y=0}^1 f(x, y)$ , we get  $f_X(0) = \frac{1}{9}$ ,  $f_X(1) = \frac{3}{9}$ ,  $f_X(2) = \frac{5}{9}$ :

$$f_X(x) = \begin{cases} 1/9, & x = 0 \\ 3/9, & x = 1 \\ 5/9, & x = 2 \\ 0, & \text{otherwise.} \end{cases}$$

(2) The marginal probability mass function  $f_Y(y)$  of  $Y$  is given below. Find the variance  $\text{Var}(Y)$ . (You may use  $E(Y) = \frac{2}{3}$ .)

$$f_Y(y) = \begin{cases} 1/3, & y = 0 \\ 2/3, & y = 1 \\ 0, & \text{otherwise.} \end{cases}$$

Solution: Using  $\text{Var}(Y) = E(Y^2) - [E(Y)]^2$  with  $E(Y) = \frac{2}{3}$ ,

$$E(Y^2) = \sum_{y=0}^1 y^2 f_Y(y) = 0^2 \cdot \frac{1}{3} + 1^2 \cdot \frac{2}{3} = \frac{2}{3}.$$

Therefore,

$$\text{Var}(Y) = E(Y^2) - [E(Y)]^2 = \frac{2}{3} - \left(\frac{2}{3}\right)^2 = \frac{2}{3} - \frac{4}{9} = \frac{6}{9} - \frac{4}{9} = \frac{2}{9}.$$

(3) Find  $E(2X - 3Y + 4)$ .

Solution: First compute  $E(X)$ :

$$E(X) = \sum_{x=0}^2 x f_X(x) = 0 \times \frac{1}{9} + 1 \times \frac{3}{9} + 2 \times \frac{5}{9} = 0 + \frac{3}{9} + \frac{10}{9} = \frac{13}{9}.$$

Using the linearity of expectation and  $E(Y) = \frac{2}{3}$ ,

$$\begin{aligned} E(2X - 3Y + 4) &= 2E(X) - 3E(Y) + 4 \\ &= 2 \times \frac{13}{9} - 3 \times \frac{2}{3} + 4 = \frac{26}{9} - \frac{18}{9} + \frac{36}{9} = \frac{44}{9}. \end{aligned}$$

(4) Determine whether  $X$  and  $Y$  are independent.

Solution: Check whether  $f(x, y) = f_X(x) f_Y(y)$  holds for all  $(x, y)$ . For  $(x, y) = (0, 0)$ ,

$$f(0, 0) = 0, \quad f_X(0) \cdot f_Y(0) = \frac{1}{9} \times \frac{1}{3} = \frac{1}{27}.$$

Since  $0 \neq \frac{1}{27}$ , we have  $f(0, 0) \neq f_X(0) f_Y(0)$ , so  $X$  and  $Y$  are **not independent**.