Introduction to Statistics - Homework #8

- Use Appendix C: Distribution tables (e.g. Z-table) from page 408 to 417 in the textbook if necessary.

Exercise 7.29 - Chicken diet and weight, Part II

Casein is a common weight gain supplement for humans. Does it have an effect on chickens? Using data provided, we test the hypothesis that the average weight of chickens that were fed casein(μ_C) is **larger** than the average weight of chickens that were fed soybean(μ_S). Assume that conditions for inference are satisfied.

(1) State the null and alternative hypothesis.

Solution:

Let μ_C and μ_S be the true mean weights of chickens fed casein and soybean, respectively.

$$H_0 : \mu_C - \mu_S = 0, \qquad H_A : \mu_C - \mu_S > 0$$

(2) Find the test statistic and its null distribution.

Solution:

Under H_0, the test statistic is:

$$T = \frac{\bar{X}_C - \bar{X}_S - 0}{\sqrt{\frac{S_C^2}{n_C} + \frac{S_S^2}{n_S}}} \sim t(df),$$

where df is the Satterthwaite's degrees of freedom.

(3) Compute the observed test statistic. The summary statistics are given below.

Diet	$\bar{x}$	s	n
Casein	323.58	64.43	12
Soybean	286.43	54.13	14

Solution:

$$t = \frac{323.58 - 286.43}{\sqrt{64.43^2/12 + 54.13^2/14}} = \frac{37.15}{23.55} \approx \boxed{1.578}$$

(4) Compute the p-value and complete the hypothesis test. (Use degrees of freedom $df = 21.63$)

$$p\text{-value} = P(T > 1.578) \approx \boxed{0.0648}$$

Since the p-value is greater than $\alpha = 0.05$, we fail to reject the null hypothesis.

Conclusion: There is not sufficient statistical evidence that the average weight of chickens that were fed casein is larger than the average weight of chickens that were fed soybean.

Exercise 7.47 - Gaming and distracted eating, Part II

A group of researchers are interested in the possible effects of distracting stimuli during eating, such as an increase or decrease in the amount of food consumption. To test this hypothesis, they monitored food intake for a group of 44 patients who were randomized into two equal groups. The treatment group ate lunch while playing solitaire, and the control group ate lunch without any added distractions. Patients in the treatment group ate 52.1 grams of biscuits, with a standard deviation of 45.1 grams, and patients in the control group ate 27.1 grams of biscuits, with a standard deviation of 26.4 grams. Do these data provide convincing evidence that the average food intake (measured in amount of biscuits consumed) is **different** for the patients in the treatment group? Assume that conditions for inference are satisfied.

(1) State the null and alternative hypotheses.

Solution: Let μ_T and μ_C be the true mean food intake (grams of biscuits) for the treatment and control groups, respectively.

$$H_0 : \mu_T - \mu_C = 0, \qquad H_A : \mu_T - \mu_C \neq 0$$

(2) Find the test statistic and its null distribution.

Solution: Under H_0, the test statistic is:

$$T = \frac{\bar{X}_T - \bar{X}_C - 0}{\sqrt{\frac{S_T^2}{n_T} + \frac{S_C^2}{n_C}}} \sim t(df),$$

where df is approximated using Satterthwaite's formula.

(3) Compute the observed test statistic. The summary statistics are given below.

Group	$\bar{x}$	s	n
Treatment	52.1	45.1	22
Control	27.1	26.4	22

Solution:

$$t = \frac{52.1 - 27.1}{\sqrt{\frac{45.1^2}{22} + \frac{26.4^2}{22}}} = \frac{25.0}{11.14} \approx \boxed{2.244}$$

(4) Find the rejection region. (Use degrees of freedom $df \approx 35.55$, rounded to $df = 36$ for the table.)

Solution: Since this is a two-sided test at significance level $\alpha = 0.05$, the critical value is $t_{0.025}(36) \approx 2.028$, so the rejection region is

$$\boxed{|t| > 2.028}, \quad \text{i.e.,} \quad t < -2.028 \quad \text{or} \quad t > 2.028.$$

(5) Complete the hypothesis test and state your conclusion in context.

Solution: Since $t \approx 2.244 > 2.030$, the observed test statistic falls in the rejection region, so we reject the null hypothesis.

Conclusion: There is statistically significant evidence that playing solitaire while eating affects the average amount of biscuits consumed.

Exercise 7.30 - Fuel efficiency of manual and automatic cars, Part II, revised

The US Environmental Protection Agency (EPA) collects fuel economy data annually. Below are summary statistics on highway fuel efficiency (MPG) from random samples of cars with automatic and manual transmissions. Conduct a hypothesis test to determine if there is a **difference** in average highway mileage between the two transmission types. **Assume that the variances of the two type's MPG are equal.**

Group	$\bar{x}$	s	n
Automatic	22.92	5.29	26
Manual	27.88	5.01	26

(1) State the null and alternative hypotheses.

Solution:

Let μ_{auto} and μ_{manual} be the true mean highway MPG for cars with automatic and manual transmissions.

$$H_0 : \mu_{\text{auto}} - \mu_{\text{manual}} = 0, \qquad H_A : \mu_{\text{auto}} - \mu_{\text{manual}} \neq 0$$

(2) Find the test statistic and its null distribution.

Solution:

Since we assume equal variances, we use the pooled standard deviation. Under H_0, the test statistic is:

$$T = \frac{\bar{X}_{\text{auto}} - \bar{X}_{\text{manual}} - 0}{S_p\sqrt{\dfrac{1}{n_{\text{auto}}} + \dfrac{1}{n_{\text{manual}}}}} = \frac{\bar{X}_{\text{auto}} - \bar{X}_{\text{manual}}}{S_p\sqrt{\dfrac{1}{26} + \dfrac{1}{26}}} \sim t(df)$$

where S_p is the pooled standard deviation and $df = n_{\text{auto}} + n_{\text{manual}} - 2 = 50$.

(3) Compute the observed test statistic using $S_p^2 = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$.

Solution:

First, calculate the pooled standard deviation:

$$s_p = \sqrt{\frac{(25)(5.29^2) + (25)(5.01^2)}{50}} \approx 5.15$$

Then, compute the test statistic:

$$t = \frac{22.92 - 27.88}{5.15 \cdot \sqrt{\dfrac{1}{26} + \dfrac{1}{26}}} = \frac{-4.96}{5.15 \cdot 0.27735} \approx \frac{-4.96}{1.428} \approx \boxed{-3.47}$$

(4) Compute the p-value and complete the hypothesis test.

Solution:

$$p\text{-value} = 2 \cdot P(T < -3.47) \approx \boxed{0.0014}$$

Since the p-value is much less than $\alpha = 0.05$, we reject the null hypothesis.

Conclusion: There is strong statistical evidence of a difference in average highway fuel efficiency between cars with automatic and manual transmissions.