

Introduction to Statistics - Homework #7

- Use Appendix C: Distribution tables (e.g. Z-table) from page 408 to 417 in the textbook if necessary.

Exercise 7.7 - Sleep habits of New Yorkers

New York is known as "the city that never sleeps." A random sample of $n = 25$ New Yorkers were asked how much sleep they get per night, yielding the following summary statistics:

$$\ast \sum_{i=1}^{25} x_i = 193.25, \quad \sum_{i=1}^{25} (x_i - \bar{x})^2 = 14.2296$$

The point estimate suggests New Yorkers sleep less than 8 hours a night on average. Is the result statistically significant? We conduct a **hypothesis test for a population mean** using a significance level of $\alpha = 0.05$. (Assume that normality condition is met.)

(a) State the null and alternative hypotheses.

(b) Find the null distribution of the test statistic.

(c) Compute the observed test statistic.

(d) Compute the p-value and complete the hypothesis test. State the conclusion in the context of the data.

Exercise: Light Bulb Lifetime with a New Filament Design

A lighting manufacturer has historically produced light bulbs whose lifetime (in hours) is normally distributed with mean $\mu_0 = 1000$ and known standard deviation $\sigma = 60$. The R&D team has developed a new filament design and suspects that the new design has **increased** the mean bulb lifetime. A random sample of $n = 36$ bulbs from the new design will be tested. We conduct a **hypothesis test for a population mean** using a significance level of $\alpha = 0.05$. (Assume the population standard deviation σ remains unchanged.)

(a) State the null and alternative hypotheses.

(b) Find the null distribution of the test statistic.

(c) Find the rejection region.

(d) Given that $\bar{x} = 1020$, complete the hypothesis test. State the conclusion in the context of the data.

(e) Assuming that the true population mean is $\mu = 1030$, compute the power of the test.

Exercise 7.20 - High School and Beyond, Part I

The National Center of Education Statistics conducted a survey of high school seniors, collecting test data on reading, writing, and several other subjects. Here we examine a simple random sample of 200 students from this survey. We want to know whether the difference between reading and writing scores is statistically significant using **paired t-test**.

(a) State the null and alternative hypotheses.

(b) Find the null distribution of the test statistic.

(c) The average observed difference in scores is $\bar{x}_{\text{diff}} = -0.545$, and the standard deviation of the differences is $s_{\text{diff}} = 8.887$. Compute the observed test statistic.

(d) Compute the p-value and complete the hypothesis test. State the conclusion in context of the data.

Exercise 7.19 - Global warming, Part I

We analyzed temperature data from 197 NOAA stations where records were available for both 1948 and 2018, comparing the number of days exceeding 90°F. The average difference (2018 - 1948) was 1.5 days with a standard deviation of 17.2 days. We seek evidence that there were more days in 2018 that exceeded 90°F from NOAA's weather stations using **paired t-test** with a significance level of $\alpha = 0.05$.

(a) State the null and alternative hypothesis. (Use a **one-sided test**.)

(b) Find the null distribution of the test statistic.

(c) Find the rejection region.

(d) Given that the sample mean of differences $\bar{x}_{\text{diff}} = 1.5$ and the sample standard deviation of differences $s_{\text{diff}} = 17.2$, complete the hypothesis test. State the conclusion in the context of the data.