

Introduction to Statistics - Homework #6

- Use Appendix C: Distribution tables (e.g. Z-table) from page 408 to 417 in the textbook if necessary.

Extra Exercise - Daily calorie intake

A researcher wants to estimate the mean daily calorie intake of adults in a neighborhood. A random sample of 36 adults from a neighborhood yields a sample mean of 2150 calories per day. Assume the population standard deviation is known to be $\sigma = 300$ calories.

Construct a 90% confidence interval for the mean daily calorie intake of adults in this neighborhood.

Solution:

We are given:

$$n = 36, \quad \bar{x} = 2150, \quad \sigma = 300$$

Since the population standard deviation σ is known, we use the standard normal (z) distribution.

For a 90% confidence interval, the critical value is $z_{0.05} = 1.645$.

The standard error is:

$$\frac{\sigma}{\sqrt{n}} = \frac{300}{\sqrt{36}} = \frac{300}{6} = 50$$

Therefore, the 90% confidence interval is:

$$\bar{x} \pm z_{0.05} \cdot \frac{\sigma}{\sqrt{n}} = 2150 \pm 1.645 \times 50 = 2150 \pm 82.25 = (2067.75, 2232.25)$$

Exercise 7.11 - Play the piano

Georgianna claims that in a small city renowned for its music school, the average child takes less than 5 years of piano lessons. We have a random sample of 20 children from the city, with a mean of 4.6 years of piano lessons and a standard deviation of 2.2 years.

Construct a 95% confidence interval for the mean number of years students in this city take piano lessons.

Solution:

We are given:

$$n = 20, \quad \bar{x} = 4.6, \quad s = 2.2$$

Since the population standard deviation σ is unknown, we use the t-distribution.

The degrees of freedom is $df = n - 1 = 19$.

Therefore, the 95% confidence interval is:

$$\bar{x} \pm t_{0.025}(19) \cdot \frac{s}{\sqrt{n}} = 4.6 \pm 2.093 \cdot \frac{2.2}{\sqrt{20}} \approx 4.6 \pm 1.030 = (3.57, 5.63)$$