Introduction to Statistics - Homework #5

- Use Appendix C: Distribution tables (e.g. Z-table) from page 408 to 417 in the textbook if necessary.

Exercise 1

Suppose we obtain a random sample $X_1, X_2, \ldots, X_n$ from a population with probability density function

$$f(x) = \begin{cases} \frac{1}{3\theta}, & -\theta < x < 2\theta,\ (\theta > 0), \\ 0, & \text{otherwise.} \end{cases}$$

Answer the following questions.

(1) Find the constant k such that the estimator $k\bar{X}$ is an unbiased estimator for θ, where $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

(2) For the value of k obtained in (1), show that $k\bar{X}$ is a consistent estimator for θ.

Exercise 2

Suppose $X_1, X_2, \ldots, X_n$ are obtained from a Bernoulli distribution $B(1, p)$. Answer the following questions.
(1) Consider the estimators

$$\hat{p}_1 = \frac{1}{n}\sum_{i=1}^{n} X_i, \qquad \hat{p}_2 = \frac{1}{2}(X_1 + X_2).$$

Show that $\hat{p}_1$ and $\hat{p}_2$ are unbiased estimator of p.

(2) Between $\hat{p}_1$ and $\hat{p}_2$, determine which estimator is more efficient (assume $n > 2$).

(3) Show that the estimator $\hat{p}_1$ is a consistent estimator for p.