Introduction to Statistics - Homework #3

Exercise 4.7 - LA Weather, Part I

The average daily high temperature in June in LA is 77°F with a standard deviation of 5°F. Suppose that the temperatures in June closely follow a normal distribution.

(1) What is the probability of observing an 83°F temperature or higher in LA during a randomly chosen day in June?

Solution: Let the random variable X be the temperature (in °F) on a randomly selected day in June. Then

$$X \sim N(77, 5^2)$$

The probability of observing an 83°F temperature or higher is:

$$P(X \geq 83) = P\left(\frac{X-77}{5} \geq \frac{83-77}{5}\right) = P(Z \geq 1.2) \approx 1 - 0.8849 = \boxed{0.1151}$$

(2) How cool are the coldest 10% of the days (days with lowest high temperature) during June in LA?

Solution: We want the 10th lower percentile of $X \sim N(77, 5^2)$. That is, find x such that $P(X \leq x) = 0.10$.

$$P(X \leq x) = P\left(\frac{X-77}{5} \leq \frac{x-77}{5}\right) = P\left(Z \leq \frac{x-77}{5}\right) = 0.10.$$

Since this the left-tail area of standard normal distribution, $\frac{x-77}{5} = -z_{0.10} \approx -1.28$

Transform back to the original scale:

$$x = 77 + (-1.28)(5) = \boxed{70.6}.$$

Exercise 4.18 - Chickenpox, Part I

Boston Children's Hospital estimates that 90% of Americans have had chickenpox by the time they reach adulthood.

(1) Calculate the probability that exactly 97 out of 100 randomly sampled American adults had chickenpox during childhood.

Solution: The number of individuals who had chickenpox follows:

$$X \sim B(100, 0.90)$$

The probability of exactly 97 individuals having had chickenpox is:

$$P(X = 97) = \binom{100}{97}(0.90)^{97}(0.10)^3 \approx \boxed{0.00589}.$$

(2) What is the probability that at least 1 out of 10 randomly sampled American adults have had chickenpox?

Solution: Define Y as the number of individuals in a sample of 10 who had chickenpox:

$$Y \sim B(10, 0.90)$$

The probability of at least one person having had chickenpox is:

$$P(Y \geq 1) = 1 - P(Y = 0)$$

where

$$P(Y = 0) = \binom{10}{0}(0.90)^0(0.10)^{10-0} = (0.10)^{10} \approx 1.0 \times 10^{-10}.$$

Thus, $P(Y \geq 1) \approx \boxed{1.0}$.

Exercise 4.20 - Chickenpox, Part II

We learned in Exercise 4.18 that about 90% of American adults had chickenpox before adulthood. We now consider a random sample of 120 American adults.

(1) How many people in this sample would you expect to have had chickenpox in their childhood? And with what standard deviation?

Solution:

The number of individuals who had chickenpox follows:

$$X \sim B(120, 0.90)$$

The expected value and standard deviation are:

$$E(X) = 120 \times 0.90 = \boxed{108}$$

$$SD(X) = \sqrt{120 \times 0.90 \times 0.10} \approx \boxed{3.29}$$

(2) What is the probability that 105 or fewer people in this sample have had chickenpox in their childhood?

Solution:

Using the normal approximation:

$$X \dot{\sim} N(108, 3.29^2)$$

Standardizing:

$$Z = \frac{X - 108}{3.29} \dot{\sim} N(0, 1)$$

Using the standard normal table:

$$P(X \leq 105) = P\left(\frac{X - 108}{3.29} \leq \frac{105 - 108}{3.29}\right) = P(Z \leq -0.91) \approx \boxed{0.181}.$$