Introduction to Statistics - Homework #2

Exercise 3.34

An airline charges the following baggage fees: $25 for the first bag and $35 for the second. Suppose 54% of passengers have no checked luggage, 34% have one piece of checked luggage, and 12% have two pieces. We assume a negligible portion of people check more than two bags.

(1) Build a probability model, compute the average revenue per passenger, and compute the corresponding standard deviation.

(2) About how much revenue should the airline expect for a flight of 120 passengers? With what standard deviation? Note any assumptions you make and if you think they are justified.

Exercise 3.47

Suppose we have n independent observations $X_1, X_2, \ldots, X_n$ from a distribution with mean μ and standard deviation σ. What is the expectation and variance of the sample mean of these n values:

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n}?$$

Extra problem 1

Suppose that X and Y have a joint probability density function (pdf)

$$f(x,y) = \begin{cases} \frac{3}{2}y^2, & 0 \le x \le 2,\ 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

and the marginal pdfs

$$f_X(x) = \begin{cases} \frac{1}{2}, & 0 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}, \quad f_Y(y) = \begin{cases} 3y^2, & 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(1) Determine if X and Y are independent.

(2) Find $E(Y)$ and $\mathrm{Var}(Y)$.

Extra problem 2

Suppose that X and Y have a joint probability mass function (pmf)

$$f(x,y) = \begin{cases} \frac{1}{12}(x+y), & x \in \{0,2\},\ y \in \{0,1,2\}, \\ 0, & \text{otherwise.} \end{cases}$$

(1) Find the marginal pmf of X, $f_X(x)$.

(2) Find the marginal pmf of Y, $f_Y(y)$.

(3) Find $E(X)$ and $E(Y)$.

(4) Find $E(XY)$ and $\mathrm{Cov}(X,Y)$.