

Introduction to Statistics - Quiz #2(60 minutes)

October 14, 2025 (Tuesday)

Section(교반): _____ Cadet Number(교번): _____ Name(성명): _____ Score: _____

- All solutions must include a detailed step-by-step explanation.
- If an answer has more than four decimal places, round to the **fourth decimal place**.
- **Reference table** is provided on the last page of the exam.

1. Jisoo's waiting time for bus X , measured in minutes, is uniformly distributed on the interval $(0, 15)$. [25 points]

(1) Find the probability density function(pdf) of X .

(2) What is the probability that Jisoo's waiting time X is less than 3 minutes?

(3) Find the expectation and variance of X .

2. Suppose the time required to complete a task follows a normal distribution with mean 30 minutes and standard deviation 3 minutes. [25 points]

(1) Find the probability that the task completion time falls between 27 minutes and 36 minutes.

(2) What is the cutoff for the top 20% of task completion times?

3. In a university survey of 800 students, 440 reported that they smoke. Construct a 95% approximate confidence interval for the population proportion of students who smoke. (Assume that conditions for the Central Limit Theorem are satisfied.) [20 points]

4. About 20% of adults worldwide drink *two or more* cups of coffee daily. To test whether the proportion of such daily coffee drinkers in South Korea differs from the global proportion $p_0 = 0.2$, a random sample of 100 adults in South Korea was surveyed. We conduct a **two-sided hypothesis test for a proportion** using a significance level of $\alpha = 0.05$. [30 points]

(1) State the null and alternative hypotheses.

(2) Find the null distribution of the test statistic. Assume that conditions for the Central Limit Theorem are satisfied.

(3) In the sample of 100 people, 28 reported drinking two or more cups daily. Compute the observed test statistic.

(4) Using the test statistic in (3), compute the p -value and complete the hypothesis test. State the conclusion in the context of data.

(5) Consider the p -value as a function of the sample proportion \hat{p} . What value of \hat{p} maximizes the p -value?

Reference Table

$z_{0.025} = 1.9600$	$z_{0.05} = 1.6449$	$z_{0.1} = 1.2816$	$z_{0.2} = 0.8416$
$P(Z \leq -2) = 0.0228, Z \sim N(0, 1)$		$P(Z \leq -1.7817) = 0.0374, Z \sim N(0, 1)$	
$P(Z \leq -1) = 0.1587, Z \sim N(0, 1)$		$P(Z \leq 0.2) = 0.5793, Z \sim N(0, 1)$	
$P(Z \leq 0.28) = 0.6103, Z \sim N(0, 1)$		$P(Z \leq 0.55) = 0.7088, Z \sim N(0, 1)$	