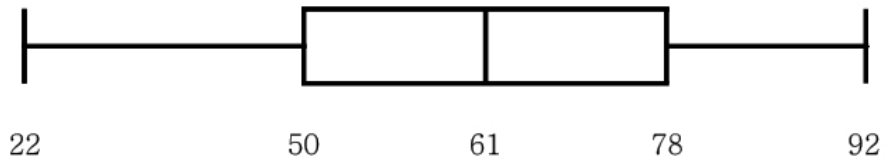
Introduction to Statistics - Quiz #1(60 minutes)

September 18, 2025 (Thursday)

Section(교반): _____ Cadet Number(교번): _____ Name(성명): _____ Score: _____

- All solutions must include a detailed step-by-step explanation.
- If an answer has more than four decimal places, round to the **fourth decimal place**.

1. A group of 100 soldiers in a training company did a sit-up test. The results are shown in the boxplot. Based on the grading standards in the table, fill in the blanks. [15 points]



Boxplot of sit-up counts for 100 soldiers

Grade	Fail	Grade 3	Grade 2	Grade 1	Special
Counts	61 or fewer	62 – 69	70 – 77	78 – 85	86 or more

Sit-up Test Grading Standards

(1) The first quartile (Q_1) of the number of sit-ups is ( ).

(2) The interquartile range (IQR) of the number of sit-ups is ( ).

(3) The number of soldiers in the company who are Grade 1 or Special grade in sit-ups is ( ).

(4) The number of sit-ups performed by the soldiers who did the fewest sit-ups in the company is ( ).

(5) The number of soldiers in the company who failed the sit-up test is ( ).

Solution: 50 / 28 / 25 / 22 / 50

2. A company uses battery packs for radios, but sometimes the batteries are defective. The probability that a battery pack is defective is 0.01. If the battery pack is defective, the probability that the radio does not work properly is 0.75. If the battery pack is not defective, the probability that the radio does not work properly is 0.02. Answer the following questions. [40 points]

(1) Find the probability that a randomly chosen radio does not work properly. (Define the relevent events before solving the problem.)

Solution: Let A be the event that the battery pack is defective, and B be the event that a radio does not work properly.

$$P(A) = 0.01,\ P(A^c) = 0.99,\ P(B \mid A) = 0.75,\ P(B \mid A^c) = 0.02.$$

It turns out that

$$\begin{aligned} P(B) &= P(A)P(B \mid A) + P(A^c)P(B \mid A^c) \\ &= (0.01)(0.75) + (0.99)(0.02) \\ &= 0.0075 + 0.0198 \\ &= 0.0273. \end{aligned}$$

(2) Find the conditional probability that the battery pack is defective, given that the radio does not work properly.

Solution: By Bayes' theorem,

$$P(A \mid B) = \frac{P(A)P(B \mid A)}{P(B)} = \frac{(0.01)(0.75)}{0.0273} = \frac{0.0075}{0.0273} \approx 0.2747.$$

3. The joint probability density function $f(x,y)$ of two random variables X and Y and the marginal probability density function $f_Y(y)$ of Y are given by

$$f(x,y)=\begin{cases} e^{-x}, & x>0,\ 0<y<1, \\ 0, & \text{otherwise.} \end{cases}, \quad f_Y(y)=\begin{cases} 1, & 0<y<1, \\ 0, & \text{otherwise.} \end{cases}$$

Answer the following questions. [45 points]

(1) Find the marginal probability density function $f_X(x)$ of X. (State the range of possible values of X).

Solution:

$$f_X(x)=\int_0^1 e^{-x}\,dy = e^{-x}, \quad x>0.$$

Therefore,

$$f_X(x)=\begin{cases} e^{-x}, & x>0, \\ 0, & \text{otherwise.} \end{cases}$$

(2) Determine whether X and Y are independent.

Solution:

Since $f(x,y)=f_X(x)f_Y(y)$, the random variables X and Y are independent.

(3) Find the expected value $E(Y)$.

Solution:

$$E(Y)=\int_0^1 y\cdot 1\,dy=\int_0^1 y\,dy=\tfrac{1}{2}.$$

(4) Find the covariance $\mathrm{Cov}(X,Y)$. You may use $E(XY)=\frac{1}{2}$ and $E(X)=1$.

Solution:

Method 1: Since X and Y are independent, and $E(X)$, $E(Y)$, and $E(XY)$ exist, $\mathrm{Cov}(X,Y)=0$.

Method 2:

$$\mathrm{Cov}(X,Y)=E[(X-E(X))(Y-E(Y))]=E(XY)-E(X)E(Y)=\tfrac{1}{2}-1\cdot\tfrac{1}{2}=0.$$

(5) Find $P(X<Y)$.

Solution:

Method 1:

$$P(X<Y)=\int_0^1\int_0^y f(x,y)\,dx\,dy=\int_0^1\int_0^y e^{-x}\,dx\,dy=\int_0^1\left(1-e^{-y}\right)dy=\left[y+e^{-y}\right]_0^1=e^{-1}.$$

Method 2:

$$P(X<Y)=\int_0^1\int_x^1 f(x,y)\,dy\,dx=\int_0^1\int_x^1 e^{-x}\,dy\,dx=\int_0^1(1-x)\,e^{-x}\,dx=\left[\left(1-e^{-x}\right)-\left(-xe^{-x}-e^{-x}\right)\right]_0^1=e^{-1}.$$