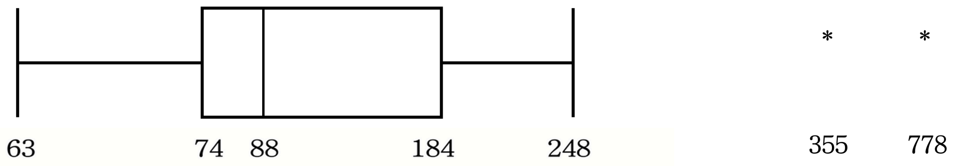
Midterm Exam

Introduction to Statistics(통계의 이해 영어강의) **2025 2nd semester**

Section(교반): ______ Cadet Number(교번): ___________ Name(성명): _______________ Score: _____________

- All solutions must include a detailed step-by-step explanation.
- If an answer has more than four decimal places, round to the **fourth decimal place**.

1.The following boxplot shows the population data (unit: thousand people) of 50 cities in a country in 1960. Fill in the blanks with the appropriate values. [10 points]



Population(unit: thousand people)

(1) The median population is ( ) thousand people.

(2) The interquartile range (IQR) of the population is ( ) thousand people.

(3) The upper limit of whiskers($Q3 + 1.5 \times IQR$) is ( ) thousand people.

(4) The number of outliers is ( ).

(5) The difference between the maximum population and the minimum population is ( ) thousand people.

Reference Table

$z_{0.005} = 2.5758$	$z_{0.01} = 2.3263$	$z_{0.025} = 1.9600$	$z_{0.05} = 1.6449$
$z_{0.1} = 1.2816$	$z_{0.15} = 1.0364$	$z_{0.2} = 0.8416$	$z_{0.25} = 0.6745$
$P(Z < -2.0000) = 0.0228,\ Z \sim N(0,1)$		$P(Z < -0.2273) = 0.4101,\ Z \sim N(0,1)$	
$P(Z > 0.0190) = 0.4924,\ Z \sim N(0,1)$		$P(Z > 0.1000) = 0.4602,\ Z \sim N(0,1)$	
$P(Z > 0.5700) = 0.2843,\ Z \sim N(0,1)$		$P(Z > 0.9000) = 0.1841,\ Z \sim N(0,1)$	
$P(Z > 1.4000) = 0.0808,\ Z \sim N(0,1)$		$P(Z > 1.5000) = 0.0668,\ Z \sim N(0,1)$	

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2. A tech company plans to launch a new project. To ensure its success, one of two external experts — A or B — will be appointed as the project director. The probabilities that A and B are appointed are 40% and 60%, respectively. If A is appointed, the probability that the project is successful is 70%; if B is appointed, the probability that the project is successful is 20%. Answer the following questions. [10 points]

(1) What is the probability that the new project is successful? (Define the relevant events before solving the problem.)

(2) If the project is successful, what is the probability that B was appointed as project director?

appoint: 임명하다; project director: 프로젝트 책임자; launch: 시작하다; expert: 전문가

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3. Alex and Ben each depart from home to a meeting point at the same time. The time taken (in minutes) for Alex is X, and for Ben is Y, and their joint probability density function is given below. [20 points]

$$f(x,y) = \begin{cases} \dfrac{1}{200}, & 35 \le x \le 55,\ 30 \le y \le 40, \\ 0, & \text{otherwise.} \end{cases}$$

(1) Find the probability that Alex arrives earlier than Ben, $P(X < Y)$.

(2) Suppose Alex and Ben make this trip $n = 30$ times. Out of $n = 30$ trips, find the expected number of times that Alex arrives earlier than Ben. (Assume that each trip is independent.)

(3) Determine whether the two random variables X and Y are independent.

(4) Find the covariance $\mathrm{Cov}(X, Y)$ of the two random variables X and Y.

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4. Boston Children's Hospital reports that $p = 90\%$ of American adults had chickenpox during childhood. Given that $n = 144$ American adults are randomly selected, let a random variable X denote the number of individuals who had chickenpox during childhood. Answer the following questions. [15 points]

(1) Calculate $P(X = 2) \times 10^{142}$.

(2) Find $E(X)$ and $Var(X)$.

(3) Find the probability that 135 or more individuals had chickenpox during childhood. Assume that the conditions for the Central Limit Theorem are satisfied.

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5. To test whether the birth rates of boys and girls are equal in a region, $n = 100$ newborns were randomly sampled from the region. We conduct **a two-sided hypothesis test for a proportion** to check whether the boys' birth rate(p) is different from $p_0 = 0.5$, using a significance level of $\alpha = 0.05$. [15 points]

(1) State the null and alternative hypotheses.

(2) Find the null distribution of the test statistic. Assume that conditions for the Central Limit Theorem are satified.

(3) In the random sample of 100 newborns, 43 were boys. Compute the observed test statistic.

(4) Compute the p-value and complete the hypothesis test. State the conclusion in the context of data.

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6. From a population having the following probability distribution, a random sample X_1 and X_2 of size $\mathbf{n}=\mathbf{2}$ is taken.

x	1	2	3
$P(X=x)$	0.6	0.2	0.2

The joint probability distribution table of the two random variables X_1 and X_2, $P(X_1=x_1, X_2=x_2)$ is given below. Answer the following questions. [15 points]

x_1 \ x_2	1	2	3
1	$0.6\times0.6=0.36$	$0.6\times0.2=0.12$	$0.6\times0.2=0.12$
2	$0.2\times0.6=0.12$	$0.2\times0.2=0.04$	$0.2\times0.2=0.04$
3	$0.2\times0.6=0.12$	$0.2\times0.2=0.04$	$0.2\times0.2=0.04$

(1) Fill in the probability distribution table of the sample mean $\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$.

$\bar{x}$						Otherwise
$P(\overline{X}=\bar{x})$						0

(2) For the sample variance $S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\overline{X})^2$, show that $S^2=\frac{1}{2}(X_1-X_2)^2$.

(3) The probability distribution table of the sample variance S^2 is given below.

s^2	0	0.5	2	Otherwise
$P(S^2=s^2)$	0.44	0.32	0.24	0

Determine whether the sample mean $\overline{X}$ and the sample variance S^2 are independent.

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7. Suppose we obtain a random sample $X_1, X_2, \dots, X_n$ from the population with probability density function

$$f(x) = \begin{cases} \dfrac{3x^2}{\theta^3}, & 0 < x < \theta \quad (\theta > 0), \\ 0, & \text{otherwise.} \end{cases}$$

For the estimator $\hat{\theta} = \dfrac{4}{3}\overline{X}_n = \dfrac{4}{3n}\sum_{i=1}^{n} X_i$, answer the following questions. [15 points]

(1) Show that the estimator $\hat{\theta}$ is an unbiased estimator of θ.

(2) Show that the estimator $\hat{\theta}$ is a consistent estimator of θ.

(3) From a random sample of size $n = 60$, the observed sample mean is $\bar{x} = 3$. Construct an approximate 95% confidence interval for θ. (Hint: Consider the sampling distribution of $\hat{\theta}$ using the Central Limit Theorem.)