Introduction to Statistics - Quiz #2(50 minutes)

April 11, 2025 (Friday)

Section(교반): _____ Cadet Number(교번): __________ Name(성명): ______________ Score: ____________

- All solutions must include a detailed step-by-step explanation.
- If an answer has more than four decimal places, round to the **fourth decimal place**.
- Reference table is provided on the last page of the exam.

1. Read the following passage and fill in the blanks with the most appropriate words from the word bank. [15 points]

Word Bank: population proportion / sample proportion / unbiased / efficient / binomial / normal / Law of Large Number / Central Limit Theorem / Type 1 error rate / Type 2 error rate / confidence level / significance level

(1) Suppose we want to estimate the population proportion using the sample proportion. In this case, the ______________________________ is the parameter, and the ________________________________ is the point estimate.

(2) The sample proportion $\hat{p}$ is an ___________ estimator of the population proportion p, because $E(\hat{p}) = p$.

(3) As the sample size increases, the sampling distribution of the sample proportion tends to follow a ___________ distribution. This phenomenon is explained by the ________________________________.

(4) In hypothesis testing, the probability of rejecting the null hypothesis when H_0 is actually true is referred to as the ________________________________ or the ________________________________.

Solution: population proportion / sample proportion / unbiased / normal / CLT / Type 1 error rate / significance level

2. At a university, final grades in a statistics course are based on a weighted average: 30% quiz (Q), 30% midterm (M), and 40% final exam (F). The quiz score Q follows a normal distribution with mean 80 and standard deviation 5. The midterm score M follows a normal distribution with mean 60 and standard deviation 20. The final exam score F follows a normal distribution with mean 70 and standard deviation 5. Assume the three scores are independent. The final weighted score W is calculated as: [35 points]

$$W = 0.3 \times Q + 0.3 \times M + 0.4 \times F$$

(1) Find the distribution of the final score W.

Solution:

$$E(W) = 0.3(80) + 0.3(60) + 0.4(70) = 24 + 18 + 28 = \boxed{70}$$

$$\mathrm{Var}(W) = 0.3^2(25) + 0.3^2(400) + 0.4^2(25) = 2.25 + 36 + 4 = \boxed{42.25}$$

$$\mathrm{SD}(W) = \sqrt{42.25} = \boxed{6.5}$$

Therefore, $W \sim N(70, 6.5^2)$, or equivalently, $W \sim N(70, 42.25)$.

(2) A student scored 90 on the quiz, 80 on the midterm, and 85 on the final exam. What proportion of students have the final score W higher than this student?

Solution:

$$W = 0.3(90) + 0.3(80) + 0.4(85) = 27 + 24 + 34 = \boxed{85}$$

$$Z = \frac{85 - 70}{6.5} = \boxed{2.3077}$$

Using the standard normal distribution table:

$$P(Z > 2.3077) \approx \boxed{0.0105}$$

Conclusion: Approximately 1.05% of students scored higher than this student.

3. In a survey, 1,000 randomly selected people were asked whether they believe it's good to consider blood type when making friends. Among them, 214 of them responded that they believe it is good to consider blood type. Construct a 95% confidence interval for the proportion of people who believe it is good to consider blood type when making friends. (Suppose that the conditions required for constructing a confidence interval for a population proportion are satisfied.) [20 points]

Solution:

We use: $z_{0.025} = 1.96, SE = \sqrt{\frac{0.214 \times (1 - 0.214)}{1000}} = 0.0130.$

The 95% confidence interval is:

$$\hat{p} \pm z_{0.025} \cdot SE = 0.214 \pm 1.96 \cdot 0.0130 \approx \boxed{(0.1886,\ 0.2394)}$$

4. It is known that approximately 10% of people worldwide have blood type B. To test whether the proportion of people with blood type B in South Korea is larger than the global proportion, a random sample of 100 people in South Korea was taken and their blood types were recorded. We conduct **a one-sided hypothesis test for a proportion** to check this, using a significance level of $\alpha = 0.05$. [30 points]

(a) State the null and alternative hypothesis.(Use **one-sided test**)

Solution:

$$H_0 : p = 0.1, \qquad H_A : p > 0.1$$

(b) Find the null distribution of the test statistic. Assume that conditions for the Central Limit Theorem are satified.

Solution:

Under H_0, the sampling distribution of $\hat{p}$ is: $\hat{p} \dot{\sim} N\left(0.1, \frac{0.1(1-0.1)}{100} = 0.03^2\right)$

or equivalently, $Z = \frac{\hat{p} - 0.1}{0.03} \dot{\sim} N(0, 1)$

(c) In the random sample of 100 people, 19 have blood type B. Compute the observed test statistic.

Solution:

$$\hat{p} = \frac{19}{100} = \boxed{0.19}, \text{ or } z = \frac{0.19 - 0.1}{0.03} = \boxed{3}$$

(d) Compute the p-value and complete the hypothesis test. State the conclusion in the context of data.

Solution:

$$p\text{-value} = P(Z > 3) \approx \boxed{0.0013}$$

Since the p-value is smaller than $\alpha = 0.05$, we reject the null hypothesis. There is strong evidence that the proportion of people with blood type B in South Korea is larger than the global proportion.

Reference Table

$z_{0.005} = 2.5758$	$z_{0.01} = 2.3263$	$z_{0.025} = 1.9600$	$z_{0.05} = 1.6449$
pnorm(0.1900, lower.tail = FALSE) = 0.4247		pnorm(1.2987, lower.tail = FALSE) = 0.0970	
pnorm(2.3077, lower.tail = FALSE) = 0.0105		pnorm(3.0000, lower.tail = FALSE) = 0.0013	