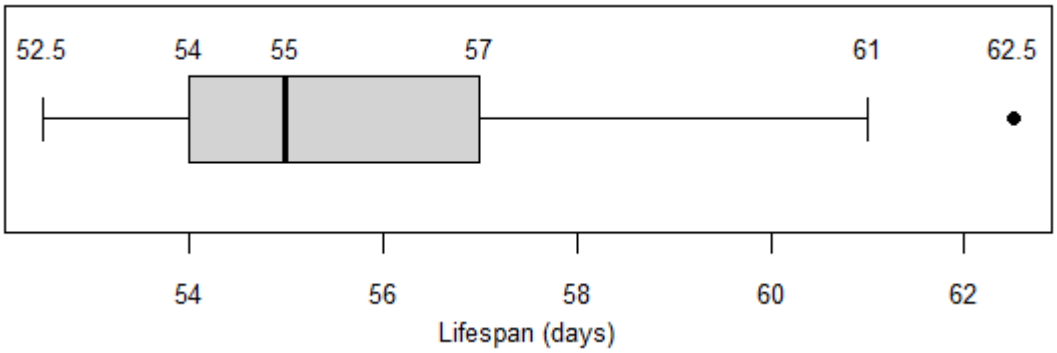
Introduction to Statistics - Quiz #1(70 minutes)

March 20, 2025 (Thursday)

Section(교반): _____ Cadet Number(교번): _____ Name(성명): _____ Score: _____

- All solutions must include a detailed step-by-step explanation.
- If an answer has more than four decimal places, round to the **fourth decimal place**.

1. A boxplot of aircraft tire lifespans produced by Company A is given below. Determine if the following statements are true or false.[10 points]



(1) The median lifespan of aircraft tires produced by Company A is 55 days. (O / X)

(2) The interquartile range (IQR) of the lifespan is 3 days. (O / X)

(3) The minimum lifespan is 54 days. (O / X)

(4) Approximately 25% of the tires have a lifespan shorter than 55 days. (O / X)

(5) The distribution of the lifespan is left-skewed. (O / X)

Solution: O / O / X / X / X

2. In a certain battalion, 0.3% of firearms are defective. If soldiers used defective firearms, 94% experienced malfunction during shooting. If soldiers used normal firearms that are not defective, 2% experienced malfucntion during shooting. Answer the following questions. [30 points]

(1) Calculate the probability that a soldier in this battalion experiences a malfunction during shooting. (Define the relevent events before solving the problem.)

Solution: Let A be the event that an issued firearm is defective, and B be the event that a malfunction occurs during shooting.

$$P(A) = 0.003, P(A^c) = 0.997, P(B|A) = 0.94, P(B|A^c) = 0.02$$

$P(B)$ can be calculated by:

$$\begin{aligned} P(B) &= P(A)P(B|A) + P(A^c)P(B|A^c) \\ &= (0.003)(0.94) + (0.997)(0.02) = 0.0228 \end{aligned}$$

(2) If a randomly selected soldier experiences a malfunction during shooting, find the probability that the soldier's firearm is defective.

Solution: Using Bayes' theorem:

$$\begin{aligned} P(A|B) &= \frac{P(A)P(B|A)}{P(B)} \\ &= \frac{(0.003 \times 0.94)}{0.0228} = 0.1237 \end{aligned}$$

lifespan: 수명, battalion: (육군)대대, firearm: 화기/총기, malfunction: 기능고장, defective: 결함이 있는

3. The joint probability density function (pdf) of two random variables X and Y and the marginal probability density function(pdf) of X are given by:

$$f(x,y)=\begin{cases}4xy, & 0<x<1, 0<y<1\\ 0, & \text{otherwise}\end{cases}, \qquad f_X(x)=\begin{cases}2x, & 0<x<1\\ 0, & \text{otherwise}\end{cases}$$

Answer the following questions. [30 points]

(1) Find the marginal probability density function of Y, $f_Y(y)$.

Solution:

$$f_Y(y)=\int_0^1 4xy\,dx=2y, \quad 0<y<1$$

$$\therefore f_Y(y)=\begin{cases}2y, & 0<y<1\\ 0, & \text{otherwise}\end{cases}$$

(2) Find the expectation of X, $E(X)$.

Solution:

$$E(X)=\int_0^1 xf_X(x)dx=\int_0^1 2x^2dx=\frac{2}{3}$$

(3) Determine whether the two random variables X and Y are independent.

Solution: Since $f(x,y)=f_X(x)f_Y(y)$, the random variables X and Y are independent.

4. A factory produces batteries with a known defect rate of 0.1. Ten batteries are randomly selected and sent to Facility P. Answer the following questions. [30 points]

(1) Let a random variable X denote the number of defective batteries among the ten sent to Facility P. Determine the probability distribution X. Also, find $E(X)$ and $Var(X)$.

Solution: $X\sim B(10,0.1)$

$E(X)=10\times 0.1=1$

$Var(X)=10\times 0.1\times 0.9=0.9$

(2) Let a random variable $Y=\dfrac{X}{10}$. Find $E(Y)$ and $Var(Y)$.

Solution: $E(Y)=\dfrac{1}{10}E(X)=0.1$

$Var(Y)=\dfrac{1}{10^2}Var(X)=0.009$

(3) Calculate the probability that at least two defective batteries are sent to Facility P.

Solution:

$$\begin{aligned}P(X\geq 2)&=1-P(X=0)-P(X=1)\\ &=1-\binom{10}{0}(0.1)^0(0.9)^{10}-\binom{10}{1}(0.1)^1(0.9)^9\\ &=1-0.3487-0.3874=0.2639\end{aligned}$$