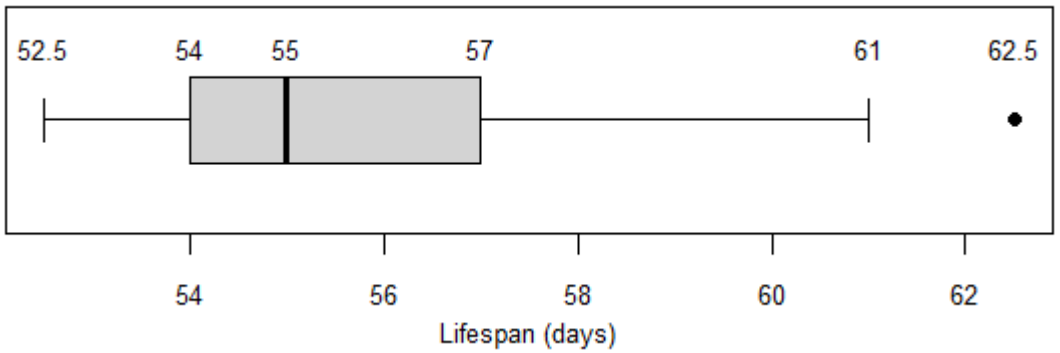
Introduction to Statistics - Quiz #1(70 minutes)

March 20, 2025 (Thursday)

Section(교반): _____ Cadet Number(교번): _____ Name(성명): _____ Score: _____

- All solutions must include a detailed step-by-step explanation.
- If an answer has more than four decimal places, round to the **fourth decimal place**.

1. A boxplot of aircraft tire lifespans produced by Company A is given below. Determine if the following statements are true or false.[10 points]



(1) The median lifespan of aircraft tires produced by Company A is 55 days. (O / X)

(2) The interquartile range (IQR) of the lifespan is 3 days. (O / X)

(3) The minimum lifespan is 54 days. (O / X)

(4) Approximately 25% of the tires have a lifespan shorter than 55 days. (O / X)

(5) The distribution of the lifespan is left-skewed. (O / X)

2. In a certain battalion, 0.3% of firearms are defective. If soldiers used defective firearms, 94% experienced malfunction during shooting. If soldiers used normal firearms that are not defective, 2% experienced malfucntion during shooting. Answer the following questions. [30 points]

(1) Calculate the probability that a soldier in this battalion experiences a malfunction during shooting. (Define the relevent events before solving the problem.)

(2) If a randomly selected soldier experiences a malfunction during shooting, find the probability that the soldier's firearm is defective.

lifespan: 수명, battalion: (육군)대대, firearm: 화기/총기, malfunction: 기능고장, defective: 결함이 있는

3. The joint probability density function (pdf) of two random variables X and Y and the marginal probability density function(pdf) of X are given by:

$$f(x,y) = \begin{cases} 4xy, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}, \qquad f_X(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Answer the following questions. [30 points]

(1) Find the marginal probability density function of Y, $f_Y(y)$.

(2) Find the expectation of X, $E(X)$.

(3) Determine whether the two random variables X and Y are independent.

4. A factory produces batteries with a known defect rate of 0.1. Ten batteries are randomly selected and sent to Facility P. Answer the following questions. [30 points]

(1) Let a random variable X denote the number of defective batteries among the ten sent to Facility P. Determine the probability distribution X. Also, find $E(X)$ and $Var(X)$.

(2) Let a random variable $Y = \dfrac{X}{10}$. Find $E(Y)$ and $Var(Y)$.

(3) Calculate the probability that at least two defective batteries are sent to Facility P.