Statistical Methods - Quiz #2(40 minutes)

September 29, 2025 (Monday)

Section(교반): __A1__ Cadet Number(교번): _____ Name(성명): _____ Score: _____

- All solutions must include a detailed step-by-step explanation.
- If an answer has more than four decimal places, round to the **fourth decimal place**.

1. Suppose that

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X.$$

Prove that

$$p(X) = \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}.$$

[25 points]

Solution: If $\log(\frac{p(X)}{1-p(X)}) = \beta_0 + \beta_1 X$, then $\frac{p(X)}{1-p(X)} = e^{\beta_0+\beta_1 X}$ and

$$\frac{1-p(X)}{p(X)} = \frac{1}{p(X)} - 1 = e^{-(\beta_0+\beta_1 X)}.$$

It follows that

$$p(X) = \frac{1}{1+e^{-(\beta_0+\beta_1 X)}} = \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}.$$

2. Suppose that an athlete has a 16% chance of getting a training injury. What is the odds that the athlete will be injured?[10 points]

Solution:

With $p = 0.16$, the odds is

$$\frac{p}{1-p} = \frac{0.16}{0.84} \approx 0.1905.$$

3. Suppose we collect data for a group of students in a statistics class with variables $X_1 =$ hours studied, $X_2 =$ undergrad GPA, and $Y =$ receive an A. We fit a logistic regression and produce the following table.[45 points]

Coefficient	Estimate	Std. Err.	Z-statistic	P-value
Intercept	-4.0000	1.2000	-3.33	0.0009
hours studied(X_1)	0.0800	0.0200	4.00	3.167×10^{-5}
undergrad GPA(X_2)	0.6000	0.3000	2.00	0.0455

(1) Estimate the probability that a student who studies for 30 hours and has an undergrad GPA of 3.2 receives an A.

Solution: The estimated probability from the logistic regression model is

$$p(X_1, X_2) = \frac{e^{\hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2}}{1 + e^{\hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2}}.$$

Substituting $X_1 = 30$, $X_2 = 3.2$,

$$p = \frac{e^{-4 + 0.08 \cdot 30 + 0.6 \cdot 3.2}}{1 + e^{-4 + 0.08 \cdot 30 + 0.6 \cdot 3.2}} = \frac{e^{0.32}}{1 + e^{0.32}} = \frac{1}{1 + e^{-0.32}} \approx 0.5793.$$

(2) How many hours would the student in part (1) need to study to have a 50% chance of getting an A in the class?

Solution: For a 50% chance, we need $p = 0.5$. This occurs when

$$\hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 = 0.$$

With $X_2 = 3.2$, this gives

$$-4 + 0.08 X_1 + 0.6 \cdot 3.2 = 0 \quad \Rightarrow \quad 0.08 X_1 - 2.08 = 0.$$

Solving yields $X_1 = 26$ hours.

(3) Is the number of hours studied (X_1) a significant predictor of receiving an A (Y)? Provide the associated p-value.

Solution: We test the hypotheses $H_0 : \beta_1 = 0$ versus $H_1 : \beta_1 \neq 0$. From the table, the p-value is 3.167×10^{-5}. Since the p-value is very small, we reject H_0 and conclude that hours studied (X_1) is a significant predictor of receiving an A.

4. Consider the following confusion matrix and answer the following questions.[20 points]

		True	
		Negative	**Positive**
Predicted	**Negative**	120	20
	Positive	15	45

(1) Compute the False Positive Rate (FPR).

Solution:

FPR = 15 / (15 + 120) = 15 / 135 ≈ 0.1111 (11.11%).

(2) Compute the False Negative Rate (FNR).

Solution:

FNR = 20 / (20 + 45) = 20 / 65 ≈ 0.3077 (30.77%).

(3) Compute the Sensitivity (True Positive Rate).

Solution:

Sensitivity = 45 / (45 + 20) = 45 / 65 ≈ 0.6923 (69.23%).

(4) Compute the Specificity (True Negative Rate).

Solution:

Specificity = 120 / (120 + 15) = 120 / 135 ≈ 0.8889 (88.89%).