

Statistical Methods - Quiz #2(40 minutes)

September 29, 2025 (Monday)

Section(교반): __A1__ Cadet Number(교번): _____ Name(성명): _____ Score: _____

- All solutions must include a detailed step-by-step explanation.
- If an answer has more than four decimal places, round to the **fourth decimal place**.

1. Suppose that

$$\log \left(\frac{p(X)}{1 - p(X)} \right) = \beta_0 + \beta_1 X.$$

Prove that

$$p(X) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}.$$

[25 points]

2. Suppose that an athlete has a 16% chance of getting a training injury. What is the odds that the athlete will be injured?[10 points]

3. Suppose we collect data for a group of students in a statistics class with variables X_1 = hours studied, X_2 = undergrad GPA, and Y = receive an A. We fit a logistic regression and produce the following table.[45 points]

Coefficient	Estimate	Std. Err.	Z-statistic	P-value
Intercept	-4.0000	1.2000	-3.33	0.0009
hours studied(X_1)	0.0800	0.0200	4.00	3.167×10^{-5}
undergrad GPA(X_2)	0.6000	0.3000	2.00	0.0455

(1) Estimate the probability that a student who studies for 30 hours and has an undergrad GPA of 3.2 receives an A.

(2) How many hours would the student in part (1) need to study to have a 50% chance of getting an A in the class?

(3) Is the number of hours studied (X_1) a significant predictor of receiving an A (Y)? Provide the associated p -value.

4. Consider the following confusion matrix and answer the following questions.[20 points]

		True	
		Negative	Positive
Predicted	Negative	120	20
	Positive	15	45

(1) Compute the False Positive Rate (FPR).

(2) Compute the False Negative Rate (FNR).

(3) Compute the Sensitivity (True Positive Rate).

(4) Compute the Specificity (True Negative Rate).